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Design of Combline and Interdigital Filters with Tapped-Line Input

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Abstract —**Explicit design equations for combline and interdigital filters with tapped-line inputs are presented. The equations are based upon a new equivalent circuit for a tapped-line input filter, derived from the open-wire-line equivalent circuit given by Cristal [1]. Using the new equivalent circuit, explicit expressions are given for all parameters of the circuit. The derivation of the design equations from the equivalent circuit is similar to that described by Matthaei *et al.* [2]. The design equations are checked by an analysis program. The results are compared to the data given by Dishal [3] and Cristal [1].**

I. Introduction

Realizations of combline and interdigital filters with tapped-line inputs have advantages over filters with the conventional transformer input [3]. Dishal [3] describes a method for tapped interdigital filter design over a narrow bandwidth. Cristal [1] provides an exact open-wire-line equivalent circuit for the tapped-line input stage. This circuit can be utilized to perform an exact analysis of the filter. For synthesis, Cristal [1] establishes an equivalence between the transformer input and tapped-line input coupling circuits and utilizes this equivalence to apply standard interdigital filter design techniques.

This method, however, has two shortcomings:

1) An exact equivalence between transformer and tapped-line filters is not possible over a broad band of frequencies [1].
2) Graphs, rather than explicit analytical expressions, are provided. The method is therefore somewhat cumbersome from a design point of view.

In this paper, the open-wire-line equivalent circuit suggested in [1] is used as a basis for another equivalent circuit. This new circuit permits the derivation of explicit expressions for the tapped-line input parameters. The subsequent design procedure is very similar to that used by Matthaei *et al.* [2] in solving the conventional problem. The validity of the design equations is checked by using the exact equivalent circuit given in [1] in an analysis program. Combline filters up to 15 percent bandwidth and interdigital filters up to an octave bandwidth are synthesized and analyzed with good agreement. These bandwidth limits are the limits of the conventional filters' design techniques. For narrow bandwidths, the expression derived for the tapping location (Φ in Fig. 1) for the interdigital filter reduces to Dishal's expression.

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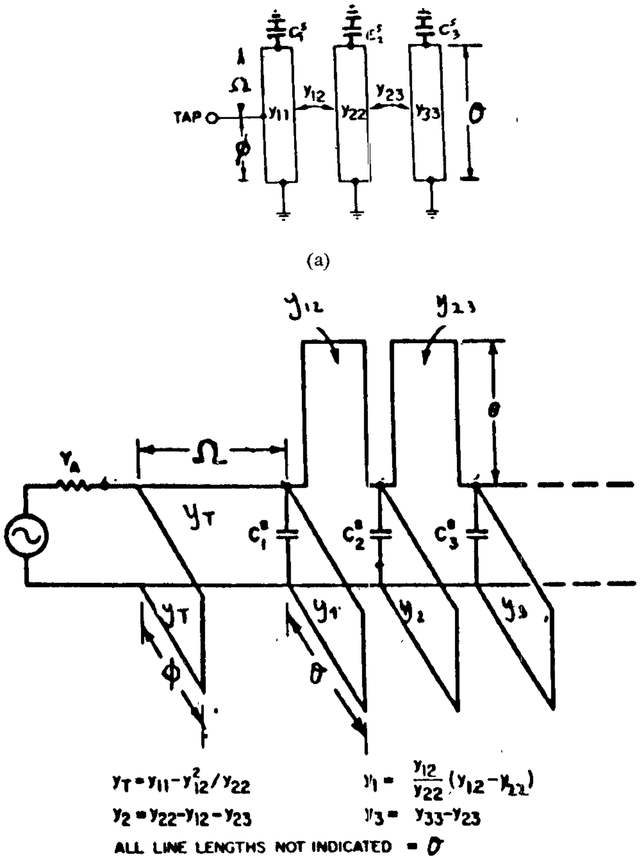

Fig. 1. Eqivalent circuit for tapped-line input combline filter. (a) Geometry and parameters. (b) Equivalent circuit.

II. Method

A. Combline Filter

The equivalent circuit suggested by Cristal [1] for the tapped-line input combline filter is shown in Fig. 1(b). The admittance inverter used to derive the equations for the combline filter [2] consists of a pi configuration, i.e., a series-shorted stub of characteristic admittance y_{12} and two shunt-shorted stubs of characteristic admittance $-y_{12}$. Therefore y_1 is split into two parts, with $y_L = (y_{12}^2/y_{22})$ added on the left to the input stage and $y_R = -y_{12}$ forming the shunt stub of the admittance inverter (J_{12}). The input stage of the filter now consists of four elements, as shown in Fig. 2.

1) New Equivalent Circuit for Input Stage: The *ABCD* matrix of the first two elements is

$$\begin{bmatrix} 1 & 0 \\ -jy_T\cot(\Phi) & 1 \end{bmatrix}\begin{bmatrix} \cos(\Omega) & jz_T\sin(\Omega) \\ jy_T\sin(\Omega) & \cos(\Omega) \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \Gamma & \delta \end{bmatrix} \tag{1}$$

where

θ electrical length of resonators,
Φ electrical length from ground to input line,
Ω $= \theta - \Phi$,
$z_T(y_T)$ impedance (admittance) of the first and second elements.

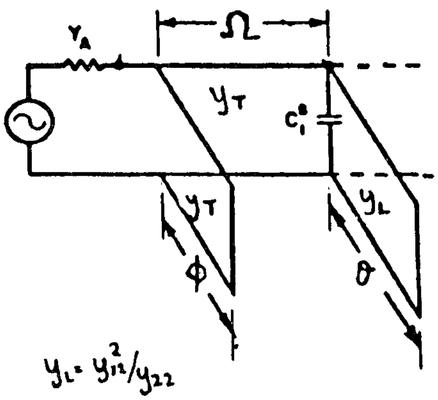

Fig. 2. Circuit of tapped-line input stage of combline filter.

The equations for $\alpha, \beta, \Gamma, \delta$ are then

$$\alpha = \cos(\Omega) \quad (2)$$

$$\beta = jz_T \sin(\Omega) \quad (3)$$

$$\Gamma = jy_T\{\sin(\Omega) - \cot(\Phi)\cos(\Omega)\} = -jy_T\cos(\theta)/\sin(\Phi) \quad (4)$$

$$\delta = \cot(\Phi)\sin(\Omega) + \cos(\Omega) = \sin(\theta)/\sin(\Phi). \quad (5)$$

Multiplying the above matrix by the $ABCD$ matrix of the other elements of the input stage (C_1^s, y_L), the $ABCD$ matrix of the input stage becomes

$$\begin{bmatrix} \alpha & \beta \\ \Gamma & \delta \end{bmatrix}\begin{bmatrix} 1 & 0 \\ y & 1 \end{bmatrix} = \begin{bmatrix} \alpha + \beta y & \beta \\ \Gamma + \delta y & \delta \end{bmatrix} \quad (6)$$

where

$$y = j\omega C_1^s - jy_L\cot(\theta). \quad (7)$$

Looking at the following equivalence:

$$\begin{bmatrix} \alpha + \beta y & \beta \\ \Gamma + \delta y & \delta \end{bmatrix} = \begin{bmatrix} 1/\delta & 0 \\ 0 & \delta \end{bmatrix}\begin{bmatrix} 1 & \beta\delta \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ \Gamma/\delta + y & 1 \end{bmatrix} \quad (8)$$

it is obvious that the first matrix represents a transformer, with a $1/\delta$ turn ratio. The second matrix represents a series impedance with an inductive reactance of $\beta\delta/j$, and the third matrix represents a shunt resonator, with the following resonance condition:

$$\Gamma/\delta + y = 0. \quad (9)$$

The series inductance must be compensated for by introducing an additional shunt capacitance C_c^s at the first resonator. This is the capacitor that Cristal [1] found necessary to introduce when comparing the tapped-input filter to the transformer input filter. The newly obtained equivalent circuit is shown in Fig. 3. Y_{a_1} is the admittance of the shunt resonator.

2) *Tapped-Line Circuit Parameters*:

a) Slope parameter: The susceptance of the tapped resonator is

$$jB = (\Gamma/\delta + y) = -j(y_T + y_L)\cot(\theta) + j\omega C_1^s \quad (10)$$

and the tapped resonator line admittance becomes

$$Y_{a_1} = y_T + y_L = y_{11}. \quad (11)$$

The susceptance slope parameter b of the tapped resonator is

$$b = \frac{\omega}{2}\cdot\frac{dB}{d\omega}\bigg|_{\omega=\omega_0} = (Y_{a_1}/2)[\theta_0/\sin^2(\theta_0) + \cot(\theta_0)] \quad (12)$$

which is the same expression as that given in [2] for a regular combline resonator.

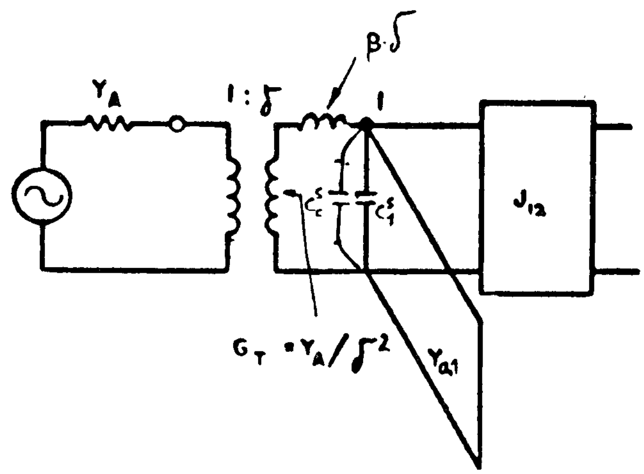

Fig. 3. New equivalent circuit.

b) Tapped input electrical length (Φ): G_T, the impedance looking back into the source impedance via the transformer, is given by

$$G_T = Y_A/(\delta^2). \quad (13)$$

From [2],

$$G_T = wb/(g_0 g_1) \quad (14)$$

where g_0, g_1 are the low-pass prototype values, and w is the fractional bandwidth. From (13) and (14),

$$Y_A/\delta^2 = wb/(g_0 g_1). \quad (15)$$

Using (5), (12), and (15), the desired equation for Φ is obtained:

$$\sin^2(\Phi_0) = Y_{a_1}\cdot w[\cos(\theta_0)\sin(\theta_0) + \theta_0]/2g_0 g_1 Y_A. \quad (16)$$

c) Lumped capacitance values: The lumped capacitance C_{1T}^s is composed of two elements: C_1^s is the same as that for the conventional combline filter and is given by

$$C_1^s = Y_{a_1}\cdot\cot(\theta_0)/\omega_0. \quad (17)$$

The additional compensating capacitance, C_c^s, is chosen to resonate the series inductive reactance $\beta\delta/j$. To first order, C_c^s is given by

$$C_c^s = Y_A^2\beta/[(\delta^3)\cdot j\omega_0].$$

Better broad-band compensation is achieved by including the interaction (at resonance) of the input and output taps:

$$C_c^s = Y_A^2\beta/[(\delta^3 - \delta\beta^2 Y_A^2)j\omega_0] \quad (18)$$

$$C_{1T}^s = C_1^s + C_c^s. \quad (19)$$

d) Distributed line capacitances: Y_{a_1}, the resonator line admittance, is chosen by the designer to fix the admittance level within the filter. From (11),

$$y_{11} = Y_{a_1}. \quad (20)$$

From the inverter circuit,

$$J_{12} = y_{12}/\tan(\theta_0).$$

From the prototype [2],

$$J_{12} = wb/(g_1 g_2)^{1/2}$$

Combining the above equations for J_{12}, y_{12} becomes

$$y_{12} = wb\cdot\tan(\theta_0)/(g_1 g_2)^{1/2}. \quad (21)$$

The normalized capacitance per unit length of the first line to ground is

$$C_1/\epsilon = 376.7/\sqrt{\epsilon_r}\cdot(y_{11} - y_{12}). \tag{22}$$

The normalized mutual capacitance per unit length between lines 1 and 2 is

$$C_{12}/\epsilon = 376.7/\sqrt{\epsilon_r}\cdot y_{12}. \tag{23}$$

A complete set of design equations for the tapped-line input combline filter is given in the Appendix.

B. Interdigital Filter

The inverter admittance used to derive the equations for the interdigital filter consists of a transmission line of characteristic admittance y_{12} and two shunt-shorted stubs of characteristic admittance $-y_{12}$. Thus the same reasoning used in the combline for splitting y_1 is valid in the interdigital case. Repeating the above procedure for an interdigital filter, the only change is that C_1^s now equals zero.

a) Tapped input electrical length (Φ): Equating all internal impedance levels of the prototype filter to h and using the transformer to match the source as derived in [4], the transformer must have a turn ratio of

$$N^2 = g_0 g_1 Y_A / h. \tag{24}$$

After frequency transformation [4] and using the scale factor of θ_1, the internal impedance of the scaled prototype becomes $h\cdot\tan(\theta_1)$, where θ_1 is the electrical length of the resonator at the low edge of the frequency passband.

Equating the scaled prototype to the equivalent circuit (Fig. 3), the following is obtained:

$$Y_{a_1} = h\cdot\tan(\theta_1) = h\cdot\tan(\pi/2\cdot(1 - w/2)) = h/\tan(\pi/4\cdot w) \tag{25}$$

$$N = 1/\delta. \tag{26}$$

Therefore (24) becomes

$$(1/\delta)^2 = g_0 g_1 Y_A / (Y_{a_1}\cdot\tan(\pi/4\cdot w)). \tag{27}$$

The formula for Φ becomes

$$\sin^2(\Phi) = \sin^2(\theta)\tan(\pi/4\cdot w)Y_{a_1}/g_0 g_1 Y_A. \tag{28}$$

The synthesis procedure for the interdigital filter, described in [4], equates the prototype to the model at the low-frequency edge of the passband. This is therefore the most accurate point of the synthesis. Solving the equation at this point, the following is obtained:

$$\Phi_1 = \sin^{-1}\left\{\left[\sin^2(\theta_1)\tan(\pi/4\cdot w)\cdot Y_{a_1}/(g_0 g_1 Y_A)\right]^{1/2}\right\} \tag{29}$$

where Φ_1 is the electrical length at the low edge of the frequency passband. For a narrow bandwidth, the following approximations are valid:

$$\tan(\pi/4\cdot w) \approx \pi/4\cdot w$$
$$\sin(\theta_1) \approx \sin(\theta_0) = 1. \tag{30}$$

Using these approximations, (29) reduces to

$$\Phi_0 = \sin^{-1}\left\{\left[\pi/4\cdot w\cdot Y_{a_1}/g_0 g_1 Y_A\right]^{1/2}\right\}. \tag{31}$$

This is the relation given by Dishal [3] for the narrow-band interdigital filter.

b) Lumped capacitance value C_c^s: The lumped capacitance value C_c^s is given by (18).

TABLE I

TYPE	w	Φ(deg)	C_T^s	C_1/ϵ	C_{12}/ϵ	Z_T	Z_1
COMBLINE	0.05	7.81	2.114	4.702	0.254	76.20	-1563
COMBLINE	0.15	13.83	2.186	4.194	0.762	77.84	-584.1
INTERDIGITAL	0.05	8.85	0.017	4.8	0.155	76.08	-2506
INTERDIGITAL	0.20	19.25	0.135	4.338	0.618	77.20	-696.0
INTERDIGITAL	0.66	44.84	0.86	2.98	1.976	90.37	-317.0

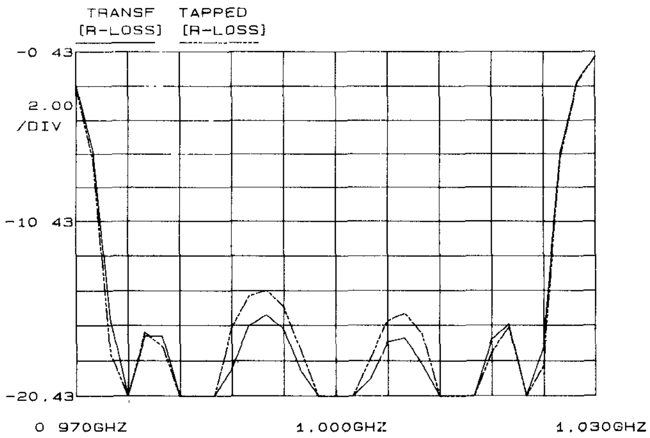

Fig. 4. Computed return loss for 5 percent bandwidth combline filter.

c) Distributed line capacitances: Y_{a_1}, the resonator line admittance, is chosen by the designer to fix the admittance level within the filter. From (11),

$$y_{11} = Y_{a_1}. \tag{32}$$

From the inverter circuit,

$$J_{12} = y_{12}/\sin(\theta_1).$$

From the low-pass prototype [4],

$$J_{12} = h/(g_1 g_2)^{1/2}.$$

Combining the above equations for J_{12}, y_{12} becomes

$$y_{12} = h\cdot\sin(\theta_1)/(g_1 g_2)^{1/2} = Y_{a_1}\cdot\cos(\theta_1)/(g_1 g_2)^{1/2}. \tag{33}$$

The normalized capacitance per unit length of the first line to ground is

$$C_1/\epsilon = 376.7/\sqrt{\epsilon_r}\cdot(y_{11} - y_{12}). \tag{34}$$

The normalized mutual capacitance per unit length between lines 1 and 2 is

$$C_{12}/\epsilon = 376.7/\sqrt{\epsilon_r}\cdot y_{12}. \tag{35}$$

A complete set of design equations for the tapped-line input interdigital filter is given in the Appendix.

III. Results

The design equations are verified by using the design procedure above and then analyzing the filters obtained by using the equivalent circuit given in [1]. All the examples are of filters with 0.1 dB ripple (16.4 dB return loss), five resonators, and an internal impedance level of 76 Ω. All the parameters of the filters are computed using the equations given in the Appendix. The parameters of the filters' input stage are summarized in Table I. Since the derivation of the design equations is based upon the methods given in [2] and [4], the same bandwidth limitations are expected. In order to compare the procedures for tapped input and transformer input filters, responses for the two filters (with

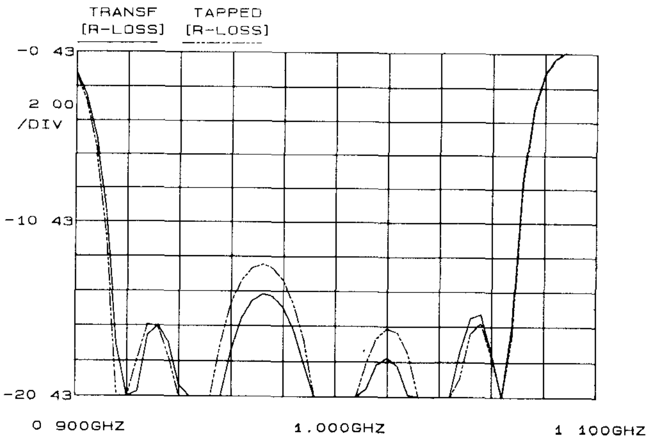

Fig. 5. Computed return loss for 15 percent bandwidth combline filter.

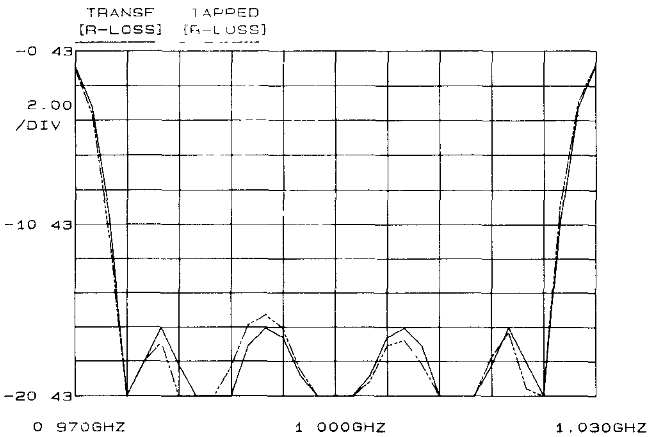

Fig. 6 Computed return loss for 5 percent bandwidth interdigital filter.

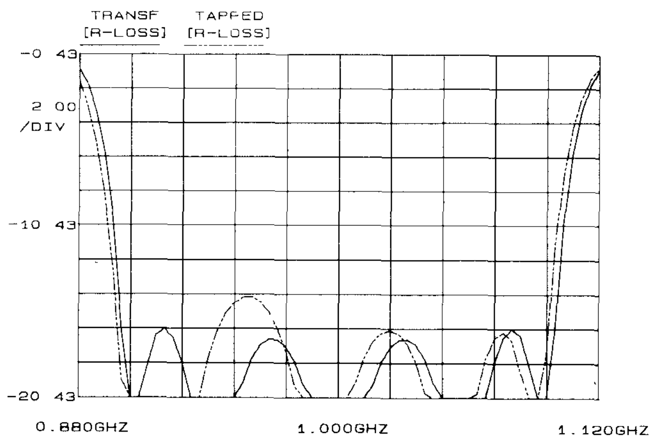

Fig 7. Computed return loss for 20 percent bandwidth interdigital filter.

the same specifications) are displayed on the same graph. Combline filters of 5 percent and 15 percent have been designed using the above procedure. The analyzed responses are plotted in Figs. 4 and 5.

Similar plots are derived for interdigital filters with 5 percent and 20 percent bandwidth, as shown in Figs. 6 and 7. The specifications of the filters are the same as those used by Cristal [1].

In Fig. 8 a design of an octave-band (66 percent) interdigital filter is shown. In this example the value of the lumped capacitor C_c^s is achieved by slight optimization (0.86 instead of 0.79). From

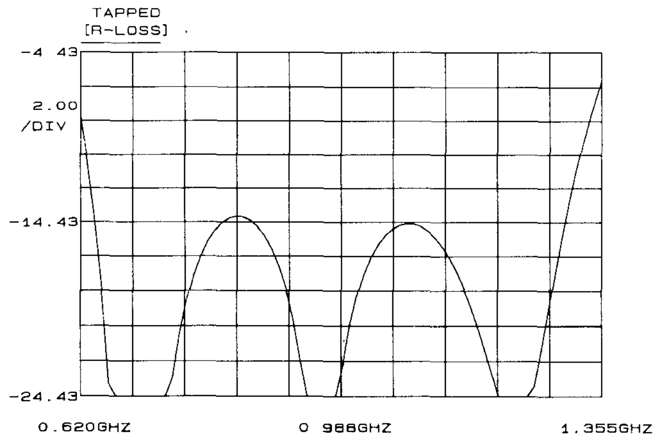

Fig. 8. Computed return loss for 66 percent bandwidth interdigital filter.

a practical point of view, it has little importance because this capacitance is achieved by means of a tuning screw.

IV. Discussion

Explicit equations are derived for tapped-line input filters. The same approximations as in the transformer input filters are used, and therefore similar responses are expected. As shown from the results, the tapped-line filter responses are very similar to the transformer filter. In the case of the octave bandwidth filter, there is a slight deviation from the desired response. This is due to the bandwidth limitation of compensation for the series impedance of $\beta\delta$. In the case of the interdigital filter, good results are achieved in comparison to previously published data. For a 20 percent bandwidth filter, the return loss of which was designed to be 16.4 dB, a 14.5 dB return loss is obtained as compared to a 12.5 dB return loss reported by Cristal [1]. For wider bandwidths there are no published data. Using this method, good results are achieved for up to an octave bandwidth.

Appendix
Design Procedure

Choose:

- N order of low-pass prototype filter,
- g_i low-pass prototype filter element values $i = 0$ to $N+1$,
- w fractional bandwidth of filter,
- Y_A source and load admittance,
- Y_{a_1} resonator line admittance (same for all resonators),
- θ_0 electrical length of resonator at center frequency, $\pi/2$ for interdigital.

Compute:

Combline

$$b = (Y_{a_1}/2)\cdot[\theta_0/\sin^2(\theta_0)+\cot(\theta_0)]$$
$$J_{j,j+1}\big|_{j=1,N-1} = wb/(g_j g_{j+1})^{1/2}$$
$$y_{j,j+1}\big|_{j=1,N-1} = J_{j,j+1}\cdot\tan(\theta_0)$$
$$\Phi_0 = \sin^{-1}\{[Y_{a_1}\cdot w(\cos\theta_0\sin\theta_0+\theta_0)/2g_0g_1Y_A]^{1/2}\}$$

Interdigital

$$\theta_1 = \pi/2\cdot(1-w/2)$$
$$h = Y_{a_1}/\tan(\theta_1)$$
$$J_{j,j+1}\big|_{j=1,N-1} = h/(g_j g_{j+1})^{1/2}$$
$$y_{j,j+1}\big|_{j=1,N-1} = J_{j,j+1}\cdot\sin(\theta_1)$$
$$\Phi_0 = \sin^{-1}\{[h\cdot\sin^2\theta_1/g_0g_1Y_A]^{1/2}\}/(1-w/2).$$

Capacitance matrix for combline and interdigital:

$$C_{j,j+1}/\epsilon|_{j=1,N-1} = 376.7/\sqrt{\epsilon_r}\cdot y_{j,j+1}$$
$$C_1/\epsilon = 376.7/\sqrt{\epsilon_r}\cdot(Y_{a_1} - y_{12})$$
$$C_j/\epsilon|_{j=2,N-1} = 376.7/\sqrt{\epsilon_r}\cdot(Y_{a_1} - y_{j-1,j} - y_{j,j+1})$$
$$C_N/\epsilon = 376.7/\sqrt{\epsilon_r}\cdot(Y_{a_1} - y_{N-1,N}).$$

Lumped capacitance:

$$C_1^s = Y_{a_1}\cdot\cot(\theta_0)/\omega_0$$
$$y_T = Y_{a_1} - y_{12}^2/Y_{a_1}$$
$$C_c^s = Y_A^2 z_T \sin(\theta_0 - \Phi_0)/\left\{\left[(\sin\theta_0/\sin\Phi_0)^3 + (\sin\theta_0/\sin\Phi_0)\cdot Y_A^2 z_T^2 \sin^2(\theta_0 - \Phi_0)\right]\omega_0\right\}.$$

Combline	Interdigital
$C_{1\,\mathrm{TOTAL}}^s = C_{N\,\mathrm{TOTAL}}^s = C_1^s + C_c^s$	$C_{1\,\mathrm{TOTAL}}^s = C_{N\,\mathrm{TOTAL}}^s = C_c^s.$
$C_j^s\|_{j=2,N-1} = C_1^s$	

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On the Realizability of the Impedance Matrix for Lossy Dielectric Posts in a Rectangular Waveguide

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***Abstract*—The equivalent T network for some lossy dielectric posts in a rectangular waveguide is found to have a negative resistance in the parallel arm although the realizability conditions for the impedance matrix are strictly satisfied. Furthermore, the reactive part of the same impedance is found to be a monotonically decreasing function of frequency. These difficulties are overcome in the case of symmetrical post structures by using lattice networks. A simplified lattice network of lumped elements is developed to approximately realize the impedance matrix for resonant lossy post structures in the bandwidth.**

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I. Introduction

A method for the analysis of dielectric posts located in a rectangular waveguide has been given in [1]. In the analysis, the dielectric posts are assumed to be uniform along the narrow side of the waveguide but are otherwise of arbitrary cross section. Furthermore, the medium of the waveguide is assumed to be linear, homogeneous, isotropic, and dissipation free. The dielectric posts are likewise linear, homogeneous, isotropic, although not necessarily free from losses. Only a few of the results for circular posts free from losses and located in a rectangular waveguide whose medium is the vacuum are reported in [1]. More results for loss-free as well as lossy posts of different configurations are given in [2].

This paper addresses the question of realizability of lossy dielectric posts in a rectangular waveguide in terms of two-port networks. Of all such networks, the T network is most commonly used. This is perhaps due to the fact that it readily realizes both symmetrical and unsymmetrical impedance matrices in a very simple and straightforward fashion. However, it is found that although the realizability conditions for the impedance matrix representation of the posts are strictly satisfied, the parallel arm impedance of the corresponding T network can have a negative real part. Furthermore, for some configurations of lossy posts, the reactive part of the same impedance is found to be a monotonically decreasing function of frequency. Such a behavior is observed for some loss-free posts as well [1], [2]. This situation, although not unlikely, does rule out any chance for obtaining a lumped network representation of resonant posts valid in the bandwidth. These lumped representations are particularly useful in the design of microwave filters employing dielectric posts in a rectangular waveguide. The purpose of this paper is therefore twofold. First, it is shown that such an irregular behavior can be avoided in the case of lossy symmetrical post structures by using lattice networks. Second, it is demonstrated that a lumped lattice network can be developed to approximately realize the impedance matrix in the bandwidth. Although the latter is accomplished by means of a working example, the procedure established is believed to be general.

II. The Realizability Problem for Lossy Posts

The realizability, and symmetry whenever applicable, conditions of the computed impedance matrix of the various configurations of lossy posts considered in [1] and [2] have always been checked. In particular, the real part $R = [R_{ij}]$, $\{i,j\} = \{1,2\}$, of the impedance matrix is found to satisfy the well-known realizability conditions [3, sec. 5-11]:

$$R_{11}, R_{22} > 0$$
$$R_{11}R_{22} - R_{12}^2 > 0. \tag{1}$$

The equivalent condition that the matrix $(U - S^H S)$, where U is the identity matrix, S is the scattering matrix of the posts, and the superscript H denotes matrix Hermitian, must be positive definite for lossy passive microwave systems [4, sec. 3-3] is also always found to be satisfied. However, in attempting to realize the impedance matrix in the form of a T network, it is found that, although the realizability conditions for the impedance matrix are strictly satisfied, the resistive part of the parallel arm impedance of the T network is negative for some post configurations. Furthermore, the reactive part of the same impedance can be a monotonically decreasing function of frequency. This situa-